PEER REVIEWED

Quantifying Shape of Lactation Curves, and Benchmark Curves for Common Dairy Breeds and Parities

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Abstract

The MilkBot® (DairySight LLC, Argyle, NY; http://milkbot.com) lactation model provides a means of quantifying both shape and magnitude of lactation curves as a set of parameter values, each of which is associated with a single aspect of lactation curve shape. Lactation data may be fitted to the model to summarize a lactation as a set of parameter values which summarize the lactation as a whole. The ***scale*** parameter controls magnitude without changing the shape of the curve, the ***ramp*** parameter controls steepness of the post-parturient rise in milk production, the ***decay*** parameter controls the rate of late lactation decline, and the ***offset*** parameter defines a theoretical offset between the start of milk production and calving. The ***decay*** parameter is easily re-expressed mathematically as ***persistence*** to quantify the rate of decline in production after peak milk. Time and quantity of peak milk, or production for any day or period in the lactation may be calculated directly from parameter values.

Aggregate normal lactation curves for mean and median milk production of Holstein, Jersey, Crossbred, Guernsey, Ayrshire, and Brown Swiss dairy cattle, stratified by parity, are calculated from a DHIA data set collected from 2005 through mid-2008 and covering over six million lactations and 51 million milk weights, mainly from farms in the eastern United States. These constitute benchmark curves that may be used as standards for normal milk production, or to quantify changes in normal productivity over time or with respect to other variables, or in econometric studies.

Keywords: lactation curves, milk production, persistence

Résumé

Le modèle MilkBot® (DairySight LLC, Argyle, NY; http://milkbot.com) de prédiction de la courbe de lactation offre la possibilité de quantifier à la fois la forme et l'ampleur des courbes de lactation en fonction d'un ensemble de paramètres qui sont chacun associés à un aspect unique de la courbe de lactation. Les données de lactation peuvent être utilisées par le modèle afin de résumer la lactation complète en une série de paramètres. Le paramètre *scale* contrôle l'ampleur sans changer la forme de la courbe; le paramètre *ramp* contrôle la pente de l'augmentation de la production laitière après le vêlage; le paramètre *decay* contrôle le taux de décroissance de la production dans la phase tardive de la lactation et le paramètre *offset* défini un décalage théorique entre le vêlage et le début de la production de lait. Le paramètre *decay* peut facilement être reformulé mathématiquement en tant que *persistence* afin de quantifier le taux de diminution de la production après le pic de production. Le moment où le pic de production est atteint et la quantité de lait produite au pic, ou à n'importe quel autre jour ou période dans la lactation, peuvent être calculés à l'aide des valeurs de ces différents paramètres.

Des courbes agrégées de lactation normale, en termes de production moyenne ou médiane et stratifiées par parité, ont été calculées chez des vaches laitières de races variées (Holstein, Jersey, Guernsey, Ayshire, Brune Suisse ou mélangée) à partir de données provenant d'un programme d'amélioration de la santé des troupeaux laitiers ramassées entre 2005 et la mi-2008 et couvrant plus de 6 millions de lactation et 51 millions pesées de lait dans des fermes situées principalement dans l'est des États-Unis. Ces courbes repères peuvent servir d'étalon pour la production normale de lait ou être utilisées pour quantifier des changements dans la production normale de lait en fonction du temps (ou de tout autre variable) ou enfin servir dans des études économétriques. D'autres manipulations mathématiques peuvent aider à calculer l'effet attendu d'une augmentation progressive vers la courbe normale sur la production totale de lait dans la lactation ou la production moyenne par jour.

Introduction

Milk production in dairy cows is a sensitive measure of cow health, as well as being the most important determinant of a dairy farm's income. Almost everything controlled in managing a dairy herd, including health, feeding, environment, breeding, and welfare, will influence daily milk production. This suggests that by developing more precise and accurate tools for measuring changes in milk production, we can better measure the influence of many other variables. This includes measuring the herd response to an intervention, the key metric of production medicine.

A lactation model can correct for the confounding effect of the normal rise then fall of production after calving, i.e. the normal lactation curve. The mathematical model, which can be customized using *a priori* information about the animals studied, can be used in many ways, particularly in computer software or spreadsheet applications to project future production or compare actual to projected production. Various curve-fitting algorithms may also be used to fit a set of observed data points to the model, yielding parameter values, which constitute a condensed quantitative summary of the data set.

Many lactation models have been proposed and compared over the past half-century,[4,5,7,8,10] but none have achieved widespread acceptance outside a few specialized applications which are directed primarily at improving estimates of actual production from incomplete data sets. The MilkBot®[a] model has close mathematical similarities to some of these earlier models, such as the Mitscherlich-Exponential model proposed by Rook[8] in 1993, but differs in that the model is derived from a theoretical-mechanistic hypothesis,[2] leading to parameters which can be interpreted both in terms of the effect that they have on the curve and in terms of the mechanistic hypothesis. This is of critical importance to the interpretability of fitted parameter values, which become metrics in their own right of the distribution of milk production within a lactation, or lactation curve shape.

A few metrics have been proposed to measure the distribution of production within a lactation that do not rely on an explicit lactation model. The term "persistency", for example, has been used to quantify the rate of decline in milk production in the later part of lactation. Unfortunately definitions of persistency vary widely,[1,6,9] and there is no standard definition in use. Similarly "peak milk", meaning the highest daily production, while strictly a measure of magnitude, can be used to quantify shape when compared to cumulative production. It can also be observed that some lactations rise more steeply after calving than others. One attempt to quantify this aspect of curve shape is "time to peak milk", which has the heavy disadvantage of being dependent on frequency of data collection (which is often monthly). Difficulties in calculation and accuracy of these measures, along with lack of clear definitions in some cases, have kept them from wide application.[3] Currently there is little understanding of normal variation in lactation curve shape between individual animals and groups aside from the very basic observation that persistency is typically higher in primiparous animals.

Materials and Methods

The MilkBot® Model (equation 1, below) predicts daily milk yield, Y(t) as a function of time (t). Four parameters, ***a*** **(*scale*)**, ***b*** **(*ramp*)**, ***c*** **(*offset*)**, and ***d*** **(*decay*)**, control the shape of the curve. The constant ***e*** is Euler's number (i.e. the root of natural logarithms, approximately 2.718).

$$1)\; Y(t) = a\,\left(1 - \frac{e^{\frac{c-t}{b}}}{2}\right) e^{-dt}$$

Interpretation of MilkBot® Parameters

Each of the parameters (a,b,c,d) in equation 1 dominates a particular aspect of lactation curve shape, and has a descriptive name related to that effect.

Parameter **a** is the ***scale*** parameter. It is a simple multiplier, which determines the overall magnitude of milk production. It can be expressed as pounds/day, kilograms/day, or similarly. This is the theoretical maximum daily milk yield, which is approached by actual peak production as ***ramp***, and ***offset*** values approach zero (i.e. a lactation which peaks on the day of calving), or as ***decay*** approaches zero (infinite ***persistence***). It must be a positive number. Changing the model to a different unit of measure for milk output only requires applying the appropriate conversion to the ***scale*** parameter, and all other parameter values remain unchanged.

Parameter **b** is the ***ramp*** parameter, controlling the rate of rise in milk production in early lactation. ***Ramp*** values are time, normally in days. Smaller ***ramp*** values imply faster creation of productive capacity and a steeper rise in early lactation. ***Ramp*** must be a positive number.

Parameter **c** is the ***offset*** parameter, and has relatively minor influence on the model. It represents the offset in time between calving and maximal growth rate of productive capacity. Its effect is slight, except in the first few days of a lactation. ***Offset*** values are time (days), and indicate the time of maximal creation of productive capacity. They may be positive, negative, or zero.

Parameter **d** is the ***decay*** parameter, controlling the loss of productive capacity, and analogous to the first-order decay constant common in pharmacokinetics. ***Decay*** is inverse-time (days^{-1}). It should be constrained to positive values under normal circumstances, though it can be argued that there might be situations in which negative ***decay*** could be biologically feasible.

A first-order decay parameter, such as the ***decay*** parameter, may easily be converted to a half-life, which allows us to address the existing concept of lactation persistency. By this re-expression (equation 2, below), the fourth parameter becomes ***persistence***, corresponding to the time it would take for production to drop by half, if we were to ignore the growth side of the model. ***Persistence*** is close to the actual half-life for productive capacity in late lactation.

$$2)\ \textbf{\textit{persistence}} = \textit{half-life of productive capacity} = {}^{0.693}/_{\textbf{\textit{decay}}}$$

Peak and Cumulative Production

Mathematical manipulation of equation 1 allows calculation of some useful results. By setting the derivative equal to zero we can calculate the day ***(tPeak)*** when milk production peaks, and then ***yPeak*** = Y (***tPeak***) gives peak milk production by substitution in equation 1.

$$3)\ t_{peak} = -b \ln\left(\frac{2db}{db+1}\right) + c$$

Cumulative production between two dates $\mathbf{t_1}$ and $\mathbf{t_2}$ can be calculated by integrating Y(t), substituting values for $\mathbf{t_1}$ and $\mathbf{t_2}$ in the integral, and finding the difference. ***M305*** is the cumulative milk yield between calving and day 305 of the lactation, calculated in this way.

$$4)\ M_{305} = \frac{a}{d}e^{-305d} + \frac{a}{2}\frac{b}{db+1}e^{\frac{c-305}{b}}e^{-305d} + \frac{a}{d} - \frac{a}{2}\frac{b}{db+1}e^{\frac{c}{b}}$$

Milk Production Data

A large set of DHIA milk production records[b] from January 2005 through June 2008 was imported into a computer database.[c] This included 51,180,569 monthly milk weights from 6,459,942 lactations in 17,013 herds. Median and mean milk weights were calculated for each days-in-milk (DIM) value between six and 400 days, for parities 1, 2, and 3 of the six most common breeds. Herds contributing records are primarily in the eastern portion of the US. Data was not otherwise edited except that records from unspecified or minor breeds was discarded.

Curve Fitting

Data for each group was fitted to the MilkBot® model using an implementation of the Levenberg-Marquardt algorithm[d] to minimize mean square error (MSE). In all cases, the fitting process converged readily and was insensitive to minor changes in input data. Fitted parameter values and MSE for each aggregate curve are reported in Table 1. An internet supplement[e] to this paper supplies high-resolution graphs showing each aggregate curve plotted with the data points to which it was fitted. Final MSE values varied between 0.32 lb (0.15 kg) and 9.9 lb (4.5 kg), depending mainly on the number of lactations in the group.

Results and Discussion

Fitted Parameter Values

Table 1 shows fitted parameter values for mean and median daily milk production for five dairy breeds, and parities 1, 2, and 3. Mean and median curves are similar in all cases, as are second and third parity lactations within each breed. There are larger differences in curve shape and magnitude between breeds and between first and later parities.

Scale rises with parity in all breeds, and varies considerably between breeds with Holsteins having greatest ***scale*** values followed by crossbred cows, then other breeds. It may be noted that the crossbred category is likely less homogeneous than the true breed categories. ***Scale*** values are a general measure of the productivity of the breed, but not tied to a particular lactation length. It remains to be seen whether ***scale*** is better or worse correlated with genetics or management than other measures of productivity, such as M305.

Ramp values seem to be lower for second parity lactations (that is a steeper rise in post-parturient production) than the other parity groups. Holstein lactations generally rise more slowly (greater ***ramp***) than other breeds. A probable but unproven hypothesis is that ***ramp*** values will be influenced heavily by transition cow management.

Offset values cover a very narrow range, but Jerseys seem to have higher ***offset*** than other breeds. Since ***offset*** represents the difference between the recorded birth of the calf and the physiological start of lactation, one possible explanation would be the excellent calving ease expected from the Jersey breed. This is because a difficult birth could delay the calving date relative to the physiological onset of lactation, which would decrease the ***offset*** value for the lactation. This is only a speculative rationalization of a small observed difference, but it seems possible that a very large database could show differences in calving ease through the ***offset*** parameter.

Decay values show the expected pattern of being considerably lower (i.e. higher ***persistence***) in lower parities. Jersey cattle seem to have the lowest ***decay*** (highest ***persistence***), and Ayrshire cattle the highest ***decay*** or lowest ***persistence***. Use of BST might have a confounding effect on ***decay*** that is difficult to predict.

Since data are aggregated before fitting, it is not possible to tell whether differences result from different distributions between breed populations, or consistent differences between individuals of different breeds. It is also possible that differences between breeds found here may be confounded by patterns of popularity of breeds in different climatic zones, or other factors, and so reflect differences in environment or management in addition to true breed characteristics.

Table 1. Fitted MilkBot® parameter values for major breeds and parities.

	Scale (lb/day)	Scale (kg/day)	Ramp (days)	Offset (days)	Decay (/day)	Persistence (days)	M305 (lb)	tPeak (days)
Median,H,P1	83.9	38.1	35.2	-3.8	0.00109	636	20,493	88
Mean ,H,P1	83.7	38.0	36.6	-3.6	0.00105	660	20,508	92
Median,H,P2	109.0	49.5	27.2	-3.8	0.00215	322	23,165	56
Mean ,H,P2	108.0	49.0	27.9	-4.0	0.00206	336	23,225	58
Median,H,P3	118.0	53.6	30.1	-2.4	0.00243	285	23,891	58
Mean ,H,P3	117.0	53.1	30.9	-2.4	0.00233	297	23,981	60
Median,X,P1	66.9	30.4	22.7	-3.4	0.00097	718	17,035	68
Mean ,X,P1	66.4	30.1	24.5	-4.7	0.00088	788	17,106	73
Median,X,P2	86.2	39.1	19.2	-2.8	0.00204	340	18,886	47
Mean ,X,P2	85.9	39.0	19.9	-3.4	0.00193	359	19,109	48
Median,X,P3	92.6	42.0	22.0	-3.7	0.00231	300	19,451	48
Mean ,X,P3	92.4	41.9	23.4	-4.2	0.00218	318	19,726	50
Median,J,P1	53.5	24.3	17.1	-2.5	0.00093	748	13,824	57
Mean ,J,P1	54.0	24.5	18.3	-2.9	0.00089	778	14,006	60
Median,J,P2	68.0	30.9	14.4	-1.6	0.00180	385	15,534	41
Mean ,J,P2	68.1	30.9	15.1	-2.0	0.00173	401	15,701	43
Median,J,P3	74.5	33.8	17.8	-1.9	0.00210	330	16,205	45
Mean ,J,P3	74.0	33.6	17.9	-2.2	0.00198	350	16,376	46
Median,G,P1	54.9	24.9	12.7	-2.3	0.00107	648	14,000	44
Mean ,G,P1	55.8	25.3	14.3	-3.3	0.00104	666	14,272	47
Median,G,P2	68.8	31.2	12.7	-2.7	0.00186	373	15,670	36
Mean ,G,P2	69.3	31.5	13.2	-3.2	0.00180	385	15,914	37
Median,G,P3	73.8	33.5	15.9	-4.2	0.00215	322	16,073	39
Mean ,G,P3	74.8	34.0	18.5	-4.9	0.00213	325	16,267	43
Median,A,P1	57.3	26.0	21.4	-5.2	0.00138	502	13,797	56
Mean ,A,P1	50.7	23.0	23.0	-7.1	0.00104	666	12,833	63
Median,A,P2	77.9	35.4	29.9	-9.0	0.00291	238	14,957	46
Mean ,A,P2	69.5	31.6	29.0	-10.8	0.00248	279	14,223	47
Median,A,P3	85.8	39.0	31.5	-7.1	0.00318	218	15,772	47
Mean ,A,P3	76.6	34.8	32.7	-9.8	0.00276	251	14,942	49
Median,B,P1	61.6	28.0	21.0	-7.2	0.00076	909	16,313	65
Mean ,B,P1	61.9	28.1	25.8	-10.5	0.00072	968	16,438	75
Median,B,P2	78.9	35.8	14.5	-5.2	0.00167	415	18,465	39
Mean ,B,P2	78.4	35.6	15.9	-6.3	0.00156	444	18,619	42

Breed code: H =Holstein, X=Crossbred, J = Jersey, G=Guernsey, A=Ayrshire, B = Brown Swiss
Parity code: P1 = first lactation, P2 = second lactation, P3 = third lactation

Persistence

Calculation of persistence as a half-life to replace current measures of persistency has advantages and disadvantages. The largest disadvantage is that it is a subtle re-expression of an existing concept, so will introduce some confusion of terms. There are already multiple conflicting definitions of persistency in use, however, some are poorly defined or applicable to only a specific section of the lactation curve. By defining a metric that is an attribute of the lactation as a whole rather than a loosely defined post-peak zone, some problems of sampling bias are avoided, and the metric benefits from a more biologically-oriented derivation.

Graphed Curves

Normal median production for Holstein, Jersey, and crossbred cattle is graphed in Figure 1 (for first parity lactations) and Figure 2 (second parity). The parameter values used in plotting the curves are taken from Table 1.

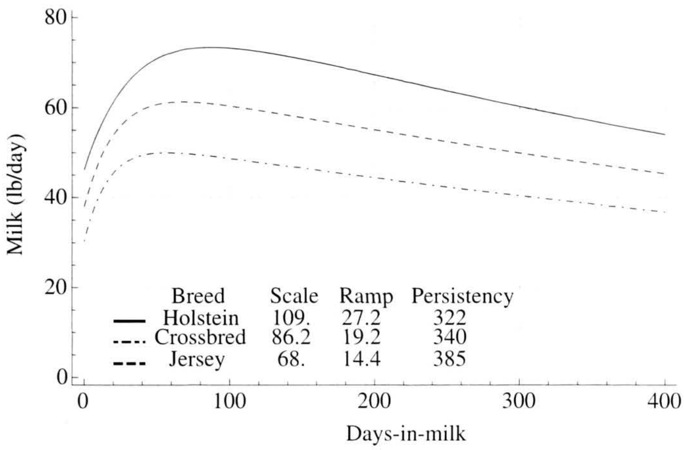

Figure 1. First parity median lactation curves.

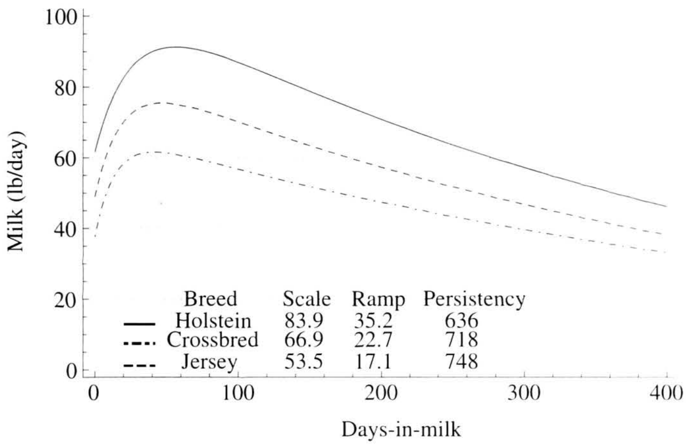

Figure 2. Second parity median lactation curves.

Figure 3 plots data points for Holstein mean daily milk along with the fitted curve, showing an excellent general fit, and also some interesting anomalies. Similar patterns are visible in graphs for other groups (not shown, but available online). There appears to be a small inflection point in the observed data just before 300 DIM, which cannot be matched by the MilkBot® mathematical model. In other words, ***persistency*** seems to improve slightly near 300 DIM. This effect is small, and could be an artifact of population dynamics rather than individual lactations. For example a subpopulation of cows with extended lactations of higher ***persistence*** would have a larger relative effect as cows in the main population become pregnant and are dried off. These hypothetical high-***persistence*** lactations might be from infertile or do-not-breed cows, and possibly the use of BST.

Also there is an anomalously low data point at 30 DIM, which may be due to data errors in reporting of calving dates. If calving dates were occasionally entered as the previous test date rather than the actual calving date, it would have the effect of biasing points near 30 DIM downwards. That is, some of the cases nominally at 30 DIM may actually be earlier in their lactations, so producing less than they would when they do reach 30 DIM. This conjecture is supported by the observation that there are about 10% more data points at 30 DIM than surrounding DIM values, suggesting about a 10% error rate for calving date.

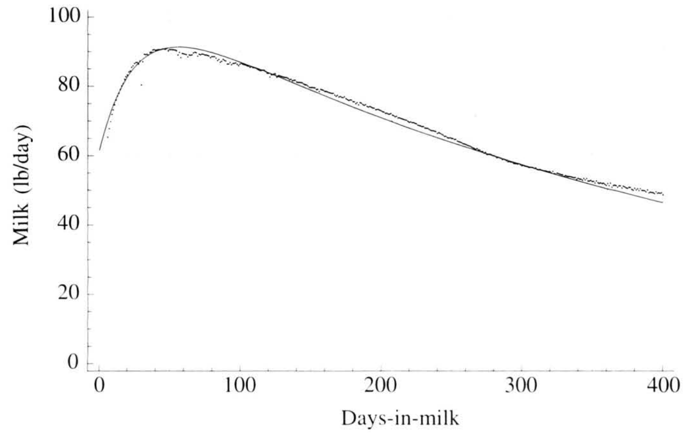

Figure 3. Holstein second parity mean daily milk with fitted lactation curve.

Benchmark Milk Yield

Parameter values reported in Table 1 can be used with the MilkBot® model to predict milk for a particular breed and parity at any value of DIM, though values for greater than 400 DIM are speculative since the model was not fitted past 400 days. For example, using parameter values for median production of second parity Holsteins [a=109 lb or 49.4 kg (***scale***), b=27.2 days (***ramp***), c= -3.8 days (***offset***), and d=0.00215 days^{-1} (***decay***)] in equation 1, we can calculate median second parity Holstein production at 100 days at 86.9 lb (39.4 kg). Similarly we can calculate that in the week between 100 DIM and 107 DIM, production is expected to drop by 1.1 lb (0.50 kg). Formulas for these calculations, formatted for use in spreadsheet software, are shown in Appendix 1. Table 1 shows that this corresponds to a 305-day total of 23,165 lb (10,507 kg). ***Persistence*** of mean milk for that group is 322 days, with a peak of 91.3 lb (41.4 kg) at DIM=56 days. This is based on 12,675,546 data points from 1,583,397 second parity Holstein lactations.

An Example Simulation

It is sometimes asserted that increasing peak milk production in a herd will increase total lactation production by some multiplier, typically around 250 lb (113 kg) of milk per incremental pound at peak. The exact increment will depend on the shape of the lactation curve, and the questionable assumption that whatever management changes altered peak milk will not alter lactation curve shape. As an example simulation, the standard curves in Table 1 were used to plot the relationship between a change in peak milk and annualized milk per cow, for several breeds and parities. Simulations of this type could easily be included in a spreadsheet or other computer software.

Total milk over a lactation of any length can be calculated similarly to the way M305 is calculated, by substituting the actual lactation length into equation 4 wherever the number 305 occurs. If that total is divided by the sum (lactation length plus dry period length), we get average milk per day during the full period of the lactation, then multiplying by 365 gives annualized milk. If this is then divided by peak milk, it can be interpreted as the expected change in annualized milk for a small incremental change in peak milk, assuming the lactation curve shape does not change. This is plotted in Figure 4, for selected breeds and parities.

The simulation indicates a higher effect for an incremental change in peak milk for parity 1 (about 285-fold) than parity 2 (about 245-fold). This could have been predicted from the flatter lactation curve of heifers. There is little difference between breeds, and the incremental effect is lower for longer lactations in cows, but minimally so for heifers. Results also depend on dry period length, which was fixed at 50 days for this simulation. This exercise is meant as an illustration of the sort of calculations that are possible from a lactation model, but not a full exploration of the scenario.

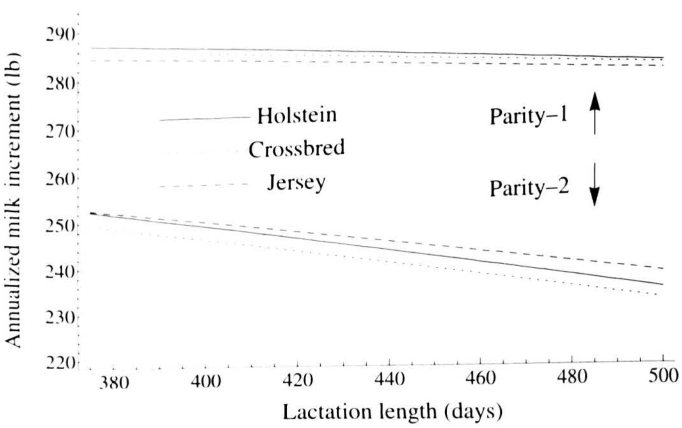

Figure 4. Effect of an incremental change in peak milk on annual milk assuming a 50-day dry period.

Internet Supplement

An internet supplement to this paper, available through the members area of the AABP website[e] includes individual graphs for each parity and breed (like Figure 3), and also graphs similar to Figures 1 and 2 for Guernsey, Brown Swiss, and Ayrshire breeds.

Conclusion

The following equation is proposed:

$$Y(t) = a\left(1 - \frac{e^{\frac{c-t}{b}}}{2}\right) e^{-dt}$$

to describe normal daily milk production as a function of DIM. Specific effects that the parameters a (***scale***), b (***ramp***), c (***offset***), and d (***decay***) have on lactation curve shape are described. The ***decay*** parameter is easily transformed into a measure of ***persistence***. The mathematical model may be used with fitting algorithms to generate quantitative measures of shape and magnitude of lactation curves.

This MilkBot® model was fitted to a large DHIA data set to generate parameter values that describe aggregate curves for major breeds and parities. These constitute benchmark curves, which are easily incorporated into computer applications or spreadsheets as standards of normal production.

MilkBot® parameters constitute a precise professional language suitable for describing lactation curve shape. With practice, it is not difficult to visualize a set of parameter values as the 2-dimensional shape of a lactation curve, or to estimate parameter values by eye from a lactation graph.

Endnotes

[a]DairySight LLC, Argyle, NY; http://milkbot.com
[b]Dairy Records Management Services, Raleigh, NC; http://drms.org
[c]MySQL, Sun Microsystems, Santa Clara, CA
[d]MilkBot® LM Fitter, DairySight LLC, Argyle, NY
[e]http://www.aabp.org/Members/publications/2011/prac_jun/EhrlichSupplement.HTM

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Appendix 1

For the convenience of readers, the MilkBot® equations are expressed here in a format that is easily transferrable into spreadsheets or other software, by replacing the named MilkBot® parameters with variables or cell references. Test values are also given. Additions or updates may also be available in the Internet Supplement.

The MilkBot® model giving daily milk as a function of scale, ramp, offset, decay, and DIM:

```
MBMilk = scale*EXP(-decay*DIM)*(1 - EXP((offset - DIM)/ramp)/2)
```

test: MBMilk(100, 20, 0, 0.002, 60) =86.4842

Persistence can easily be calculated from decay, and vice versa.

```
persistence = 0.693/decay
decay = 0.693/persistence
```

test: persistence(.002) =346.5
test: decay(346.5) = .002

Time of peak milk and Peak milk:

```
PeakDay = =offset - ramp *LN((2*ramp*decay)/(1+ramp*decay))
```

```
PeakMilk = scale*EXP(-decay*(offset - ramp *LN((2*ramp*decay)/(1+ramp*decay))))*(1 - EXP((offset - (offset - ramp *LN((2*ramp*decay)/(1+ramp*decay))))/ramp
```

or

```
PeakMilk = MBMilk(scale, ramp, offset, persistence, PeakDay(scale, ramp, offset, persistence)
```

test: PeakDay(100, 20, 0, 0.002) = 51.299
test: PeakMilk(100, 20, 0, 0.002) = 86.7779

The formulas for calculation of M305, or total milk between two days (start, stop):

```
M305 =((1-EXP(-305*decay))*scale)/decay+(EXP(offset/ramp)*((-1+EXP(305*(-decay-1/ramp)))*ramp*scale)/(2+2*decay*ramp))

CumMilk =((EXP(-decay*start)-EXP(-decay*stop))*scale)/decay+(EXP(offset/ramp)*(-EXP(start*(-decay-1/ramp))+EXP(stop*(-decay-1/ramp)))*ramp*scale)/(2+2*decay*ramp)
```

test: M305(100, 20, 0, .002) = 21870.9
test: cumMilk(100, 20, 0, .002, 0, 305) = 21870.9

Users of Microsoft Excel may want to implement these as VBA UDFs. An example is shown below.

```
Public Function MBMilk(mbScale As Double, mbRamp As Double, mbOffset As Double, mbDecay As Double, mbDay As Double) As Double
MBMilk = mbScale * Exp(-mbDecay * mbDay) * (1 - Exp((mbOffset - mbDay) / mbRamp) / 2)
End Function


Public Function MBPeakDay(mbScale As Double, mbRamp As Double, mbOffset As Double, mbDecay As Double) As Long
MBPeakDay = Round(mbOffset - mbRamp * Log((2 * mbRamp * mbDecay) / (1 + mbRamp * mbDecay)))
End Function


Public Function MBPeakMilk(mbScale As Double, mbRamp As Double, mbOffset As Double, mbDecay As Double, mbDay As Double) As Double
MBPeakMilk = MBMilk(mbScale, mbRamp, mbOffset, mbDecay, MBPeakDay(mbScale, mbRamp, mbOffset, mbDecay))
End Function


Public Function MBPersistence(mbDecay As Double) As Double
MBPersistence = 0.693 / mbDecay
End Function

Public Function MTT(mbScale As Double, mbRamp As Double, mbOffset As Double, mbDecay As Double) As Double
MTT = ((1 - Exp(-305 * mbDecay)) * mbScale) / mbDecay + (Exp(mbOffset / mbRamp) * ((-1 + Exp(305 * (-mbDecay - 1 / mbRamp))) * mbRamp * mbScale) / (2 + 2 * mbDecay * mbRamp))
End Function
```